Antitrust in Innovative Industries*

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PRELIMINARY AND INCOMPLETE

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1. Introduction

Much has been written about the increasing importance of intellectual property, and industries based upon it, in the U.S. economy. Even ignoring the recent "dot-com bubble," the list of the largest firms in the U.S. economy is increasingly dominated by firms operating in industries in which innovation is central to a firm's success. Competition in such "new economy" industries is often said to possess characteristics that are quite different from those of traditional "old economy" industries. As Evans and Schmalensee [2002] succinctly put it,

> "...firms engage in dynamic competition *for the market* — usually through research-and development (R&D) to develop the 'killer' product, service, or feature that will confer market leadership and thus diminish or eliminate actual or potential rivals. Static price/output competition on the margin *in the market* is less important."

In the wake of these changes, and sparked by the recent *Microsoft* case, a number of commentators have expressed the concern that traditional antitrust analysis — which has typically ignored almost entirely issues of innovation — might be poorly suited to maximizing welfare in such industries.[1]

In the *Microsoft* case, for example, the most significant issue in evaluating the welfare effects of Microsoft's allegedly anticompetitive practices was almost surely their effect on innovation. In essence, Microsoft argued that while a technological leader like Microsoft may possess a good deal of static market power, this is merely the fuel for stimulating dynamic competition, a process that it argued worked well in the industry. The government, in contrast, argued that Microsoft's practices prevented entry of new firms and products, and therefore would both raise prices and retard innovation. (For further discussion, see e.g., Evans and Schmalensee [2002] and Whinston [2001].) How to reconcile

[1]For an example of such an argument, see again Evans and Schmalensee [2002]. Issues of innovation have been considered when discussing "innovation markets" in some horizontal merger cases, where there was a concern that a merger might reduce R&D competition. See, e.g., Gilbert and Sunshine [1995].

these two views, however, was never fully clear in the discussion surrounding the case. For example, if profits are necessary for spurring innovation as Microsoft argued, does this mean that practices that enhance a dominant firm's ability to protect its monopoly position will spur innovation?[2]

In this paper, we study the role of antitrust policy in innovative ("dynamically competitive") industries. We do so using models in which innovation is a continual process, with new innovators replacing current incumbents, and holding dominant market positions until they are themselves replaced. Although a great deal of formal modeling of R&D races has occurred in the industrial organization literature (beginning with the work of Loury [1979] and Lee and Wilde [1980]; see Reinganum [1989] for a survey), this work has typically analyzed a single, or at most a finite sequence, of innovative races.[3] Instead, our models are closer to those that have received attention in the recent literature on growth (e.g., Grossman and Helpman [1991], Aghion and Howitt [1992], Aghion et. al [2001]). The primary distinction between our analysis and the analysis in this growth literature lies in our explicit focus on how antitrust policies affect equilibrium in such industries.[4]

The paper is organized as follows. In Section 2, we introduce and analyze a simple stylized model of antitrust in an innovative industry. This simple model, in which only

[2]Note that there is a potentially important distinction here between a policy that restricts Microsoft's behavior and a policy that restricts the behavior of all dominant software producers. The former type of policy is sure to increase the likelihood of success of today's potential entrants. However, the relevant question concerns the latter type of policy, which may not increase innovative activity, because today's potential entrants are spurred precisely by the hope of becoming the next Microsoft.

[3]One exception is O'Donohue et al. [1998] who use a continuing innovation model to examine optimal patent policy. In Section 3.5 we discuss the relation of our analysis to their paper.

[4]The growth literature often considers how changes in various parameters will affect the rate of innovation, sometimes even calling such parameters measures of the degree of antitrust policy (e.g., Aghion et al. [2001] refer to the elasticity of substitution as such a measure). Here we are much more explicit than is the growth literature about what antitrust policies toward specific practices do. This is not a minor difference, as our results differ substantially from those that might be inferred from the parameter changes considered in the growth literature. As one example, one would get exactly the wrong conclusion if one extrapolated results showing that more inelastic demand functions lead to more R&D (e.g., Aghion and Howitt [1992]) to mean that allowing an incumbent to enhance its market power through long-term contracts leads to more R&D.

potential entrants conduct R&D, captures antitrust policy as affecting the profit flows that an incumbent and a new entrant can earn in competition with each other, as well as the profits of an uncontested incumbent. Using the model, we develop some general insights into the effect of antitrust policies on the rate of innovation. We show that a more protective antitrust policy (one that increases a new entrant's profits at the expense of the incumbent) "front-loads" an innovative new entrant's profit stream, and that this feature tends to increase the level of innovative activity by potential entrants to the industry. Indeed, as long as a more protective policy dissipates neither the joint profit of the incumbent and entrant upon entry nor uncontested incumbent profits, it will increase the level of R&D. We also explore extensions of the model to situations of free entry, to growing markets, and to predatory activities that affect an entrant's probability of survival.

With the stylized model of Section 2 in hand, in Section 3 we develop applications to specific antitrust polices. First, we study a model of long-term (exclusive) contracts and show that a more protective antitrust policy necessarily stimulates innovation and raises both aggregate and consumer welfare. Next, we study a model of predatory pricing. Once again, a more protective policy necessarily stimulates R&D in our model, although we show that the welfare implications are in general ambiguous. We also discuss voluntary deals between the incumbent and entrant, such as licensing deals or collusive agreements, which can also be seen to necessarily increase the rate of innovation. All three of these applications have the feature that a more protective policy (one that enhances entrant profits) increases the rate of innovation. We conclude Section 3 by discussing an extension of our long-term contracting model to the case of uncertain innovation size with a fixed cost of implementing new innovations. We show that in this situation, a more protective policy may retard innovation. The key new feature in this model is that the antitrust policy has a "selection effect," altering the set of innovations that enter the market. In some cases, this factor can lead a more protective policy to reduce innovation and welfare.

The analysis of Sections 2 and 3 makes the strong assumption that only potential entrants do R&D. While useful for gaining understanding, this assumption is rarely descriptive of reality. In Section 4 (still incomplete), we turn our attention to models in which both incumbents and potential entrants conduct R&D. Introducing incumbent investment has the potential to substantially complicate our analysis by making equilibrium behavior depend on the level of the incumbent's lead over other firms. We study two models in which we can avoid this state dependence. In one model, the previous leading technology is assumed to enter the public domain whenever the incumbent innovates. In this model, the incumbent does R&D solely to avoid displacement by a rival. In our second model, the profit improvement from a larger lead is assumed to be linear in the size of the lead and potential entrants are assumed to win all "ties," which again leads the incumbent's optimal R&D level to be stationary. In this model, the incumbent does R&D to improve its profit flows until the time that it is displaced by a rival. Interestingly, we show that in both models there are a wide range of circumstances in which a more protective policy can increase the innovation incentives of both the incumbent and potential entrants.

Finally, Section 5 (to be added) concludes.

2. A Simple Model of Antitrust in Innovative Industries

We begin by considering a simple stylized model of continuing innovation. Our aim in this section is to develop a model that yields some general insights into the effect of antitrust policies on the rate of innovation, and that we can apply to a number of different antitrust policies in the remainder of the paper. The model has discrete time and an infinite horizon. There are $N+1$ firms who discount future profits at rate $\delta \in (0,1)$. In each period, one of the firms is the "incumbent" I and the others are "potential entrants," denoted collectively by E. In the beginning of each period, each potential entrant i independently chooses its R&D rate, $\phi_i \in [0,1]$; the cost of R&D is given by the convex function $c(\phi_i)$.[5] (Note that in this simple model only the potential entrants may do R&D; we relax this assumption to consider incumbent investment in Section 4). The R&D of a given potential entrant i yields an innovation — which we interpret to be a particular improvement in the quality of the product — with probability ϕ_i. We shall focus on symmetric equilibria, in which all potential entrants choose the same equilibrium level of R&D, denoted by ϕ. In this case, the likelihood that at least one firm among the N potential entrants innovates is given by $s(\phi, N) \equiv \left[1-(1-\phi)^N\right]$. Among the potential entrants who discover the innovation, only one may receive the patent for that innovation. Given that all other potential entrants are doing R&D at level ϕ, we denote by $r(\phi, N)$ the probability that a given potential entrant receives a patent, conditional on it making a discovery.[6] A potential entrant who is successful at receiving a patent enters and competes with the incumbent in the present period, and then becomes the incumbent in the next period, while the previous incumbent then becomes a potential entrant. In this sense, this is a model of "winner-take-all" competition. While the patent provides perfect protection (forever) to the innovation itself, others may overtake the patent holder by developing subsequent innovations.

We will be interested in the effects of an antitrust policy α that affects the incumbent's competition with an entrant who has just received a patent. To this end, we denote the incumbent's profit in competition with a new entrant by $\pi_I(\alpha)$, and the profit of the entrant by $\pi_E(\alpha)$, which we assume are differentiable functions of α. We let $\pi'_E(\alpha) > 0$, so that a higher α represents a policy that is more "protective" of the entrant. We also denote by the differentiable function $\pi_m(\alpha)$ the per period profit of an incumbent who

[5] Note that $c(\cdot)$ must be convex if firms can randomize over their R&D strategies.

[6] When the patent is awarded randomly to one of the successful innovators, we have

$$r(\phi, N) = \sum_{k=0}^{N-1} \left(\frac{1}{k+1}\right) \binom{N-1}{k} \phi^k (1-\phi)^{N-1-k}.$$

It should be noted, however, that the results of this section hold for any functions $r(\phi, N)$ and $s(\phi, N)$; for example, there may be some probability that none of the firms that have made discoveries are successful in commercializing its product.

faces no competition. (In Section 3, when we consider specific applications, we show how these values can be derived from an underlying model of the product market.) We assume that

$$\pi_E(\alpha) + \left(\frac{\delta}{1-\delta}\right)\pi_m(\alpha) > c'(0) \tag{A1}$$

and

$$\left(\frac{\pi_E(\alpha) + \delta\pi_I(\alpha)}{N+\delta}\right) < c'(1). \tag{A2}$$

Assumption (A1) will imply that it is worth doing a little R&D if no one else is doing any R&D; it ensures a positive level of R&D in equilibrium. Assumption (A2) will imply that the probability of successful innovation is below 1 in equilibrium.

We examine stationary Markov perfect equilibria of the infinite-horizon game using the dynamic programming approach. Let V_I denote the expected present discounted profits of an incumbent, and V_E those of a potential entrant (both evaluated in the beginning of a period). Then, since innovation occurs with probability ϕ, these values should satisfy

$$V_I = \pi_m(\alpha) + \delta V_I + s(\phi, N)\left[\pi_I(\alpha) - \pi_m(\alpha) + \delta(V_E - V_I)\right], \tag{VI}$$

$$V_E = \delta V_E + \phi r(\phi, N)\left[\pi_E(\alpha) + \delta(V_I - V_E)\right] - c(\phi). \tag{VE}$$

Also, since a potential entrant's choice of ϕ should maximize its expected discounted value given that all other potential entrants are choosing R&D level ϕ,

$$\phi \in \arg\max_{\psi\in[0,1]} \left\{\psi r(\phi, N)\left[\pi_E(\alpha) + \delta(V_I - V_E)\right] - c(\psi)\right\}.$$

Letting $W \equiv r(\phi, N)[\pi_E(\alpha) + \delta(V_I - V_E)]$ denote the expected benefit from successful innovation — what we shall call the **innovation prize** — this equation can be rewritten

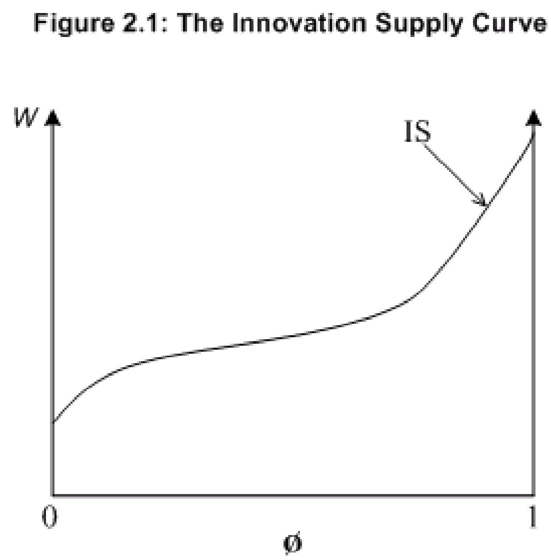

Figure 2.1:

as

$$\phi \in \arg\max_{\psi \in [0,1]} \{\psi W - c(\psi)\}. \tag{IS}$$

Note that the convexity of $c(\cdot)$ implies that the set of maximizers in (IS) is a non-empty and convex set.

This equation defines the "Innovation Supply" curve—the optimal innovation choice as a function of W. Note that this curve, which we depict in Figure 2.1, is (weakly) upward sloping by the Monotone Selection Theorem (Milgrom and Shannon [1994]).

Consider now the determinants of W. Subtracting (VE) from (VI), we can write [to simplify notation, we suppress the arguments of $s(\phi, N)$ and $r(\phi, N)$]:

$$(V_I - V_E) = \frac{s\pi_I(\alpha) + (1-s)\pi_m(\alpha) - \phi r \pi_E(\alpha) + c(\phi)}{1 - \delta + \delta(s + \phi r)}. \tag{2.1}$$

Since $W \equiv r(\phi, N)[\pi_E(\alpha) + \delta(V_I - V_E)]$, substituting from (2.1) lets us solve for the

Figure 2.2: Equilibrium and Comparative Statics

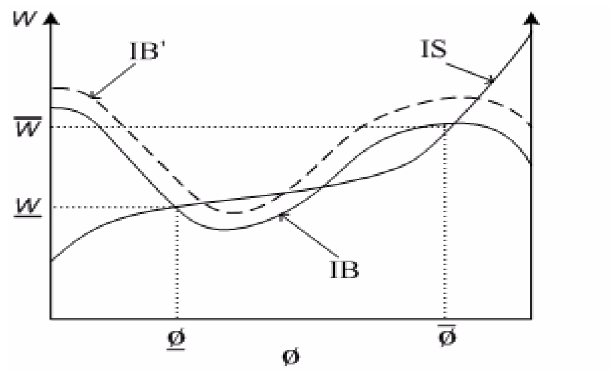

Figure 2.2:

equilibrium value of the innovation prize W:

$$
\begin{aligned}
W &= r\left\{\pi_E(\alpha)+\delta\left[\frac{s\pi_I(\alpha)+(1-s)\pi_m(\alpha)-\phi r\pi_E(\alpha)+c(\phi)}{1-\delta+\delta(s+\phi r)}\right]\right\} \\
&= r\left\{\frac{\pi_E(\alpha)\{1-\delta+\delta[s+\phi r]\}+\delta[s\pi_I(\alpha)+(1-s)\pi_m(\alpha)-\phi r\pi_E(\alpha)+c(\phi)]}{1-\delta+\delta(s+\phi r)}\right\} \\
&= r\left\{\frac{\pi_E(\alpha)(1-\delta)+\delta\{s[\pi_I(\alpha)+\pi_E(\alpha)]+[1-s]\pi_m(\alpha)+c(\phi)\}}{1-\delta+\delta(s+\phi r)}\right\}. \qquad \text{(IB)}
\end{aligned}
$$

This equation defines the "Innovation Benefit" curve — the value of the innovation prize as a function of the per firm innovation rate ϕ. An equilibrium pair (W, ϕ) must lie at an intersection of (IS) and (IB), as shown in Figure 2.2 where there are three equilibria.

Note that the (IS) curve does not depend on α at all. By Theorem 1 of Milgrom and Roberts [1994], if α shifts the (IB) curve up or down at all values of ϕ, then it increases or reduces the equilibrium innovation rate in the "largest" and "smallest" equilibria (denoted by $\underline{\phi}$ and $\overline{\phi}$ respectively in Figure 2.2). This can be seen in Figure 2.2, where the dashed curve represents an upward shift of the IB curve. As is also evident in the figure, the same can be shown (using the Implicit Function Theorem) of any "stable" equilibrium if the IB function is shifted up or down in a neighborhood of the equilibrium.[7]

Differentiating (IB) with respect to α then yields the following result:

Proposition 2.1. *Under assumptions (A1) and (A2), an increase in α, the protectiveness of antitrust policy, increases (decreases) the rate of innovation in the equilibria with the highest and lowest innovation rates if*

$$\pi_E'(\alpha) + \delta\left[\frac{(1-s)\pi_m'(\alpha) + s\pi_I'(\alpha)}{1-\delta(1-s)}\right] \geq (\leq)\ 0 \tag{2.2}$$

at all feasible s.[8] *Moreover, the change in a stable equilibrium's innovation rate in response to a local change in α is positive (negative) if and only if (2.2) holds at the equilibrium level of s.*

Condition (2.2) indicates that a change in policy encourages (discourages) innovation precisely when it raises (reduces) the incremental expected discounted profits over an innovation's lifetime: The first term on the left side of (2.2) is the change in an entrant's profit in the period of entry due to the policy change, while the second term is equal to the change in the value of a continuing incumbent (the numerator is the derivative of the flow of expected profits in each period of incumbency conditional on still being an incumbent; the denominator captures the "effective" discount rate, which includes the probability of displacement), and thus of the entrant's value once it is itself established as the incumbent.

In interpreting condition (2.2), it is also helpful to think about the case in which the monopoly profit π_m is independent of α, so that $\pi_m'(\alpha) = 0$. In this case, condition (2.2) tells us that the innovation rate ϕ increases if

$$\pi_E'(\alpha) + \left[\frac{\delta s}{1-\delta(1-s)}\right]\pi_I'(\alpha) \geq 0.$$

[7] When there is a unique equilibrium, this result then implies determinate comparative statics. Equilibrium can be shown to be unique, for example, when $N = 1$, $c'(\phi) \geq 0$, and $c''(\phi) > 0$.

[8] *The set of feasible s is $\{s : s = s(\phi, N)$ for some $\phi \in [0,1]\}$.*

Thus, when $\pi_m'(\alpha) = 0$, innovation increases if a weighted sum of $\pi_E(\alpha)$ and $\pi_I(\alpha)$ increases, where the weight on $\pi_E(\alpha)$ exceeds the weight on $\pi_I(\alpha)$ due to discounting ($\delta < 1$). This implies that a more protective antitrust policy raises innovation whenever $\pi_I'(\alpha) + \pi_E'(\alpha) \geq 0$; that is, provided that an increase in α does not lower the joint profits of the entrant and the incumbent in the period of entry. Intuitively, observe that a successful innovator earns $\pi_E(\alpha)$ when he enters, and earns $\pi_I(\alpha)$ when he is displaced. A more protective antitrust policy that raises π_E and lowers π_I shifts profits forward in time. Since the later profits π_I are discounted by potential entrants this "front loading" of profits necessarily increases the innovation prize provided that the joint profit $\pi_I + \pi_E$ does not decrease.

Observe also that the weight on $\pi_I'(\alpha)$ is increasing in s and in δ. The larger is δ or s, the more likely is a more protective policy to reduce innovation: For s, this is so because larger s moves forward the expected date when the entrant will itself be replaced. For δ, this is so because with larger δ the discounted value of the profits in the period in which the entrant is replaced are greater. In the limit, as $\delta \to 1$, the amount by which the joint profit $\pi_E + \pi_I$ can be dissipated while still encouraging innovation converges to zero: in this limiting case, the cost of a one dollar reduction in the value π_I that the entrant will receive when he is ultimately displaced is exactly equal to the gain from receiving a dollar more in the period in which he enters.

2.1. More general profit functions

In general, all three of the profits $\pi_I, \pi_E,$ and π_m may be affected as well by the rate of innovation s (this is true, for example, in the model of long-term contracts in Section 3.2). Denoting these profits by $\pi_I(\alpha, s)$, $\pi_E(\alpha, s)$, and $\pi_m(\alpha, s)$, we see that the derivation leading to Proposition 2.1 continues to hold in this case, since it involves asking when the IB curve is shifted upward at a particular value of s. Thus, we need only reinterpret the derivatives in (2.2) as being partial derivatives with respect to α holding s fixed.

2.2. Free entry

In some circumstances it may be more appropriate to assume that there is free entry into R&D competition.[9] This assumption can be interpreted as a limiting case in which the number N of potential entrants engaging in R&D is very large. We still focus on the symmetric equilibrium in which all entrants choose the same R&D level $\phi \in (0,1)$, but in the limit where N is large, each entrant chooses a positive but infinitesimal ϕ. The first-order condition for such infinitesimal innovation choice is

$$W = c'(0). \tag{2.3}$$

Also, substituting $\phi = c(\phi) = 0$ and writing r as a function of s in (IB), we have[10]

$$W = r(s)\left\{\frac{\pi_E(\alpha)(1-\delta) + \delta\{s[\pi_I(\alpha) + \pi_E(\alpha)] + (1-s)\pi_m(\alpha)]\}}{1-\delta+\delta s}\right\}. \tag{2.4}$$

The two equations describe the IS and IB curves in (W, s) space, which are depicted in Figure 2.3 (note that the IS curve is horizontal).

Differentiating expression (2.4) with respect to α, we see that an increase in α increases the aggregate success rate s if and only if (2.2) holds; that is, under exactly the same conditions as when N is fixed.

[9] The fixed N model is the appropriate model when there are a limited number of firms with the capability of doing R&D in an industry (perhaps because of some complementary assets they possess due to participation in related industries).

[10] Specifically, if the patent is awarded with equal probability among those firms that make a discovery, we have

$$r(s) = s + \sum\nolimits_{k=1}^{\infty}\left(\frac{1}{k+1}\right)\left(\frac{s}{1+s}\right)^k.$$

Figure 2.3: Equilibrium and Comparative Statics with Free Entry

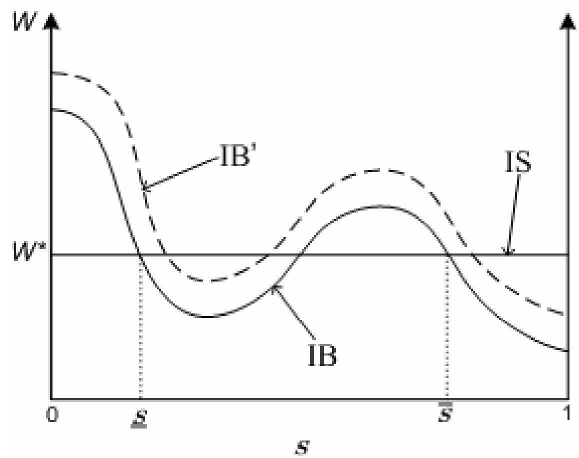

Figure 2.3:

2.3. Market growth

Above we noted how the "front loading" of the profits from successful innovation caused by a more protective antitrust policy raises the innovation prize, and hence the equilibrium rate of innovation. This same logic suggests, however, that in situations in which market size is growing rapidly — so that the future looms large relative to the present — such front-loading may no longer encourage innovation. To see this point, consider the simplest possible case of market growth, in which the profit functions in period 1 are $\beta\pi_E(\alpha)$, $\beta\pi_I(\alpha)$, and $\beta\pi_m$, but are $\pi_E(\alpha)$, $\pi_I(\alpha)$, and $\pi_m(\alpha)$ beginning in period 2. The market is initially growing if $\beta < 1$.

Starting in period 2, the market is stationary, and the equilibrium values and innovation rate are exactly those derived above. Denote these, as before, as V_E, V_I, W, and ϕ. Denoting the R&D level of each potential entrant in period 1 by ϕ_1 and letting $r_1 \equiv r(\phi_1, N)$, the period 1 innovation prize W_1 is given by

$$W_1 = r_1[\beta\pi_E(\alpha) + \delta(V_1 - V_0)].$$

By analogy with condition (2.2), the policy change increases the rate of innovation in period 1 if and only if

$$\beta\pi_E'(\alpha) + \delta\left[\frac{(1-s)\pi_m'(\alpha) + s\pi_I'(\alpha)}{1-\delta(1-s)}\right] \geq 0,$$

since only the profits in period 1 are affected by the market growth term β.[11] Thus, with market growth ($\beta < 1$), an increase in α that has no effect on π_m may lower the rate of innovation during the growth phase even if it raises the joint profit $\pi_I + \pi_E$.[12]

2.4. Predatory activities

In the analysis to this point, antitrust policy altered the profits earned by the incumbent and the entrant when entry occurs, π_I and π_E, and possibly the profits of an uncontested incumbent π_m. In some situations, antitrust may affect as well the entrant's probability of survival. Here we focus solely on this effect. Specifically, we take π_I, π_E, and π_m as fixed and suppose that a new entrant's probability of survival following its entry is $\lambda(\alpha)$ where $\lambda(\cdot)$ is increasing in α. As before, we focus here on the case in which the number of firms N is fixed.

Now the innovation prize is

$$W = r[\pi^E + \delta\lambda(\alpha)(V_I - V_E)]. \tag{2.5}$$

If $(V_I - V_E)$ were fixed, an increase in α would necessarily increase innovation. Now,

[11] This expression can alternatively be derived by solving explicitly for W_1.

[12] Note that we have focused to this point on stationary antitrust policies. The present result suggests that it may be of interest to consider policies that vary over time with market conditions, such as the current market size or growth rate.

$$V_I = \pi_m + \delta V_I + s(\phi, N)\left[\pi_I - \pi_m + \delta\lambda(\alpha)\left(V_E - V_I\right)\right], \tag{VI-2}$$

$$V_E = \delta V_E + \phi r(\phi, N)\left[\pi_E + \delta\lambda(\alpha)\left(V_I - V_E\right)\right] - c\left(\phi\right), \tag{VE-2}$$

so that

$$V_I - V_E = \frac{s\pi_I + (1-s)\,\pi_m - \phi r \pi_E + c(\phi)}{1 - \delta + \delta\lambda(\alpha)(s + \phi r)}. \tag{2.6}$$

We see then that an increase in α lowers $(V_I - V_E)$. Substituting, however, gives

$$W = r\left\{\pi_E + \left(\frac{\delta\lambda(\alpha)}{1 - \delta + \delta\lambda(\alpha)(s + \phi r)}\right)\left[s\pi_I + (1-s)\,\pi_m - \phi r \pi_E + c(\phi)\right]\right\}. \tag{2.7}$$

The fraction $\delta\lambda(\alpha)/[1-\delta+\delta\lambda(\alpha)(s+\phi r)]$ is increasing in α. Thus, we see from 2.6 that W is increasing in α provided that $(V_I - V_E)$ is positive.[13] Hence, provided that $(V_I - V_E)$ is positive, a more protective antitrust policy that raises the likelihood of entrant survival necessarily increases the innovation prize. As can be seen in (2.5), this change would clearly increase W if we were to hold $(V_I - V_E)$ fixed. Yet, even though $(V_I - V_E)$ is affected by the change (larger α lowers the difference in values between an incumbent and a potential entrant), the net effect is still necessarily positive; the increased chance of becoming a continuing incumbent today more than compensates for the increased chance of being displaced tomorrow.

3. Applications

In this section, we study several models of antitrust policy toward specific practices as an application of the results of Section 2. The models are all versions of the "quality

[13] For example, this will always be true whenever $V_E = 0$ (say, because of a constant returns to scale R&D technology) and π_m and π_I are non-negative.

ladder" models introduced in the recent literature on economic growth (e.g., Aghion and Howitt [1992]; Grossman and Helpman [1991]). Before turning to these applications, we first introduce a basic quality ladder model (in which antitrust policy plays no role) to serve as a benchmark.

3.1. A quality ladder model

There are $N+1$ firms and a continuum of infinitely-lived consumers of measure 1 who may consume a nonstorable and nondurable good with production cost $c \geq 0$. R&D may improve the quality of this good and consumers value "generation j" of the good at $v_j = v + j \cdot \Delta$. At any time t, one firm — the current "incumbent" — possesses a perfectly effective and infinitely-lived patent on the latest generation product j_t. Likewise, at time t there is a patentholder for each of the previous generations of the product ($j_t - 1, j_t - 2, ...$). We assume, as in Section 2, that at time t only firms other than the incumbent in the leading technology — the potential entrants — can invest in developing the generation $j_t + 1$ product. One implication of this assumption is that in each period t the holder of the patent on generation $j_t - 1$ is a firm other than the current incumbent, who holds the patent on the current leading generation j_t. We assume that at time t, the firms engage in Bertrand competition to make sales. Thus, $\pi_E = \pi_m = \Delta$ and $\pi_I = 0$.[14] Specializing (IB) to this case we have

$$
\begin{aligned}
W &= r\left[\frac{\Delta(1-\delta) + \delta[s\Delta + (1-s)\Delta + c(\phi)]}{1-\delta+\delta(s+\phi r)}\right] \\
&= r\left[\frac{\Delta + \delta c(\phi)}{1-\delta+\delta(s+\phi r)}\right]. \qquad (3.1)
\end{aligned}
$$

As before, the equilibrium innovation rate ϕ satisfies $\phi \in \arg\max_{\psi \in [0,1]} \{\psi W - c(\psi)\}$.

Since we now have a fully-specified consumer side (unlike in Section 2), we can com-

[14] We focus here on the undominated equilibrium in which the incumbent (who makes no sales) charges a price equal to cost and the entrant with technology $j_t + 1$ charges a price of Δ.

pare the equilibrium innovation rate to the rate that maximizes aggregate welfare. To this end, observe that a technological advancement in period t raises gross consumer surplus in every subsequent period by Δ. The present discounted value of this change is $\left(\frac{\Delta}{1-\delta}\right)$. A firm who innovates is critical for advancing the technology in that period if and only if no other firm has successfully innovated. Thus, if the socially optimal (symmetric) innovation rate is ϕ°, the "Social Innovation Prize" is given by

$$W_S = (1-\phi^\circ)^{N-1}\left(\frac{\Delta}{1-\delta}\right), \tag{3.2}$$

which defines a downward-sloping Social Innovation Benefit Curve. The socially optimal innovation rate ϕ° must lie at an intersection of this Social Innovation Benefit curve and the Innovation Supply Curve depicted in Figure 2.1. Since the Innovation Supply Curve is (weakly) upward sloping, there is at most a single intersection. Thus, the relation between ϕ° and ϕ can be determined by the relation between W and W_S.

In general, the equilibrium level of innovation may be either higher or lower than the level that maximizes social surplus because W may be either higher or lower than W_S. This is due to two distortions: First, there is a "Schumpeterian effect" because an innovator is eventually replaced even though his innovation raises surplus indefinitely. To see this effect, it is useful to define the value of a new patent to be

$$P \equiv \left[\frac{\Delta + \delta c(\phi)}{1-\delta+\delta(s+\phi r)}\right].$$

Doing so, we see that the private innovation prize is $W = rP$. Now note that $V_E \geq 0$ if and only if $\phi r P - c(\phi) \geq 0$, which implies using (3.1) that

$$P \leq \frac{\Delta}{1-\delta+\delta s}; \tag{3.3}$$

in any equilibrium with $V_E \geq 0$.[15] Thus, we see that $P \leq \frac{\Delta}{1-\delta}$, so that the value of a new patent never exceeds the social value of a technological advancement, $\frac{\Delta}{1-\delta}$. On the other hand, a "business stealing effect" is also present, since a potential entrant is sure to get a patent when all other firms have failed, but also gets the patent in some cases when another firm has succeeded; only in the latter case, however, has the innovation contributed to social surplus. This effect is captured by the fact that, when the patent is awarded randomly to one of the innovators, $r \geq (1-\phi)^{N-1}$.[16] Note that when $N = 1$ only the former effect is present [since $r = 1 = (1-\phi)^0$], in which case the equilibrium rate of R&D is less than the level that maximizes social surplus. Likewise, as $\delta \to 1$, the socially optimal R&D rate $\phi^\circ \to 1$, while the equilibrium level is bounded below this level provided that assumption (A2) holds with $\delta = 1$. On the other hand, if $\delta \to 0$ and $N > 1$, the equilibrium rate will exceed ϕ°.

3.2. Long-term (exclusive) contracts

We now consider a model in which the incumbent can sign consumers to long-term contracts. We normalize the total number of consumers in each period to 1. Suppose that in each period t, the incumbent can offer long-term contracts to a share β_{t+1} of period $t+1$ consumers. The contracts specify a sale in period $t+1$ at a price q_{t+1} to be paid upon delivery. (In our simple model, this is equivalent to an exclusive contract that prevents the consumer from buying from the entrant, subject to some irrelevant issues with the timing of payments.) The antitrust policy restricts the proportion of consumers

[15] In particular, since $\phi r P \geq c(\phi)$ when $V_E \geq 0$, we have

$$P = \frac{W}{r} \leq \left(\frac{1}{r}\right) r \left[\frac{\Delta + \delta\phi r P}{1 - \delta + \delta(s + \phi r)}\right]$$

which implies (3.3).

[16] In Aghion and Howitt [1992], two additional distortions are present: an "appropriability effect" (an incumbent monopolist captures less than his full incremental contribution to social surplus in a period) and a "monopoly distortion" effect (an incumbent produces less than the socially optimal quantity in each period). These two distortions are absent here because of our assumption of homogeneous consumer valuations and Bertrand competition.

that can be offered long-term contracts: $\beta_{t+1} \leq 1 - \alpha$. We assume that the production cost c exceeds the quality increment Δ, so that an entrant cannot profitably make a sale to a customer who is bound to a long-term contract.

The timing in period t is:

- **Stage** t.**1:** Each potential entrant i observes the share of captured customers β_t and chooses its innovation rate ϕ_{it}. Then innovation success is realized.

- **Stage** t.**2:** Firms name prices p_t^i to free period t consumers

- **Stage** t.**3:** Free period t consumers accept/reject these offers.

- **Stage** t.**4:** The firm with the leading technology chooses to offer to a share $\beta_{t+1} \leq 1 - \alpha$ of period $t+1$ consumers a period $t+1$ sales contract at price q_{t+1} to be paid upon delivery.

- **Stage** t.**5:** Period $t+1$ consumers accept/reject these contract offers (they assume that they have no effect on the likelihood of future entry).[17]

We look at Markov perfect equilibria. In particular, we focus on equilibria in which potential entrants in stage $t.1$ condition their innovation choices only on the current share of captive customers β_t, and in which the choices at all other stages are stationary. (Note that since period t contracts expire at the end of that period, there is no relevant state variable affecting the contracting choice of the leading firm at stage $t.4$.)

It is immediate that in any such equilibrium, the prices offered to free customers in any period t are $c + \Delta$ by the firm with the leading technology j_t, who wins the sale, and c by the firm with technology $j_t - 1$. Now consider a consumer's decision of whether to accept a long-term contract. If in period t the expected innovation rate in period $t+1$ is s_{t+1}, a period $t+1$ consumer who rejects the leading firm's long-term

[17] We assume throughout that consumers all accept if accepting is a continuation equilibrium (we do not allow consumers to coordinate). The leading firm could achieve this by, for example, committing to auction off the desired number of long-term contracts.

contract offer anticipates getting the period t surplus level $\overline{v} + (j_t - 1)\,\Delta - c$ plus an expected gain in surplus of $s_{t+1}\Delta$ due to the possibility of technological advancement in period $t+1$. Thus, he will accept the contract if and only if the price q_{t+1} satisfies $\overline{v} + j_t\Delta - q_{t+1} \geq \overline{v} + (j_t - 1 + s_{t+1})\,\Delta - c$. Hence, the maximum price the incumbent can receive in a long-term contract is $q_{t+1} = c + (1 - s_{t+1})\Delta$, which leaves the consumer indifferent about signing.

How many consumers will the leading firm sign up in period t? Observe first that if the aggregate innovation rate s_{t+1} were independent of β_{t+1}, then the leading firm would be indifferent about signing up an extra consumer: its period t expectation of the profit from a free consumer in period $t+1$ is $(1 - s_{t+1})\Delta$, which exactly equals its maximal expected profit from a long-term contract. However, each entrant's optimal innovation choice ϕ_{t+1} is decreasing in β_{t+1}, because it reduces the profits a successful entrant can collect in period $t+1$.[18] Therefore, the incumbent will sign up as many long-term customers as the antitrust constraint allows, i.e., $\beta_{t+1} = 1 - \alpha$ in every period. This implies, in particular, that the equilibrium innovation rate s_t is also stationary. We can therefore fit this model into our basic model by taking

$$\begin{aligned} \pi_m(\alpha, s) &= \alpha\Delta + (1-\alpha)(1-s)\Delta \\ \pi_I(\alpha, s) &= (1-\alpha)(1-s)\Delta \\ \pi_E(\alpha, s) &= \alpha\Delta. \end{aligned} \tag{3.4}$$

Observe, first, that in this model an increase in α does indeed raise π_E. More signifi-

[18] Formally, the equilibrium innovation rate of a potential entrant in period $t+1$ satisfies $\phi_{t+1} \in \arg\max_{\psi \in 0[,1]} \psi r\left(\phi_{t+1}, N\right)\left[(1-\beta_{t+1})\Delta + \delta\left(V_I^{t+2} - V_E^{t+2}\right)\right]$, where V_I^{t+2}, V_E^{t+2} are the continuation values at the start of period $t+2$ which are independent of β_{t+1}. Since $r\left(\phi_{t+1}, N\right)$ is decreasing in ϕ_{t+1}, there is a unique solution to this fixed-point problem, and by Theorem 1 of Milgrom and Roberts [1994], this solution is decreasing in β_{t+1}.

cantly,

$$[s\pi_I(\alpha, s) + (1-s)\pi_m(\alpha, s)] = (1-s)\alpha\Delta + (1-\alpha)(1-s)\Delta = (1-s)\Delta.$$

Thus, holding ϕ fixed, the expected profit of a continuing incumbent is independent of α. Thus, we see immediately that condition (2.2) is satisfied, and so we have:

Proposition 3.1. *In our basic model of long-term (exclusive) contracts, restricting the use of long-term contracts encourages innovation.*

An alternative way of seeing the result is to observe that an increase in α raises both the joint profit upon entry $\pi_E + \pi_I$, as well as the profit of an uncontested monopolist π_m, and so must raise innovation. Joint profits upon entry are increased because the incumbent has had to offer a discount below a price of Δ to induce the captured consumers to sign (he is getting them to agree to buy a worse product than the entrant's with probability s). Uncontested monopoly profits are increased by an increase in α because, given that entry has not occurred, the incumbent is better off the fewer consumers it has signed up at discounted prices.

Consider now the welfare effects of a once-and-for-all increase in the policy α. Note, first, that the increase raises consumer surplus: consumers are indifferent about signing exclusives when the innovation rate is held fixed, but an increase in the innovation rate delivers to them higher-quality goods at the same prices. The current incumbent is hurt by the change: it would not be affected if the innovation rate were held fixed, but it is hurt by the increase in the innovation rate which speeds its replacement. What about the sum of consumer surplus and current incumbent profits? Intuitively, an innovation in period t reallocates surplus $\alpha\Delta$ from the incumbent to period t consumers. However, in subsequent periods the innovation confers an expected benefit Δ to consumers but at an expected cost to the incumbent that is less than Δ as long as the probability of future displacement is positive (i.e., $s > 0$). Finally, consider the effects on the potential entrants. Since we are staying on the upward-sloping IS curve, the increase in ϕ caused by the increase in α increases W. This implies that each potential entrant becomes better off. This reasoning leads to the following result:

Proposition 3.2. *A once-and-for-all restriction on the share of long-term (exclusive) contracts raises consumer surplus, the profits of potential entrants, and aggregate welfare. It reduces the current incumbent's profits.*

Proof. We consider in turn the change in the payoffs of entrants, the current incumbent, consumers, and the current incumbent plus consumers.

Potential Entrants: If innovation increases W must have increased. Using (VE) and (IS), we see that

$$(1-\delta)\, V_E = \max_{\psi \in [0,1]} \left\{ \psi r(\phi, N) W - c\,(\psi) \right\},$$

which implies that a potential entrant's value V_E has increased.

Current Incumbent: We first argue that the current incumbent's value falls. To see this, observe from (VI) that we can write

$$\begin{aligned}
(1-\delta)V_I &= [(1-s)\pi_m + s\pi_I] - s\delta(V_I - V_E) \\
&= [(1-s)\pi_m + s\pi_I] - s\left[\frac{W}{r} - \pi_E\right] \\
&= \Delta - s(1-\alpha)\Delta - s\left(\frac{W}{r}\right),
\end{aligned}$$

which decreases with the change in α since s and W increase, and r decreases.

We next compute a convenient lower bound on the value change of the continuing incumbent. A policy change at the start of some period τ changes the current incumbent's profits only beginning in the next period. The current incumbent's continuation payoff in the next period is $\delta[s_0 V_E + (1-s_0)V_I]$, where s_0 is the innovation rate before the policy change (i.e., the probability of innovation in period τ) and V_E and V_I are the values after the policy change (i.e., at the start of period $\tau+1$). Since V_E has gone up, a lower bound on the change in the incumbent's payoff is the change in $\delta(1-s_0)V_I$. From equation (VI) we see that we can write

$$\begin{aligned}(1-\delta+\delta s)V_I &= [(1-s)\pi_m + s\pi_I] + \delta s V_E \\ &= [(1-\alpha)(1-s)\Delta + (1-s)\alpha\Delta] + \delta s V_E \\ &= (1-s)\Delta + \delta s V_E.\end{aligned}$$

Again, since V_E has increased, a lower bound on the change in the current incumbent's payoff is the change in

$$\frac{\delta(1-s_0)(1-s)\Delta}{(1-\delta+\delta s)}. \tag{3.5}$$

Consumers: Consumer welfare does not change until period $\tau+1$ either. Since every consumer is always indifferent between signing an exclusive and being free, we can derive consumer welfare from period $\tau+1$ on by assuming that all consumers are free. Thus consumer welfare starting in period $\tau+1$ is

$$\delta[(v_{j_\tau}+s_0\Delta-c-\Delta)+s\frac{\Delta}{1-\delta}]+\delta^2[(v_{j_\tau}+s_0\Delta-c-\Delta)+s\frac{\Delta}{1-\delta}]+... = \delta[(v_{j_\tau}-c-\Delta)+s\frac{\Delta}{(1-\delta)^2}], \tag{3.6}$$

where v_τ is the value of the quality of the leading good at the start of period τ.

Sum of Current Incumbent and Consumers: Adding (3.5) and (3.6) a lower bound on the change in the sum of consumer plus current incumbent payoffs is given by the change in

$$\frac{\delta(1-s_0)(1-s)\Delta}{(1-\delta+\delta s)} + \delta s\frac{\Delta}{(1-\delta)^2},$$

which is increasing in s. ■

It is perhaps surprising that the welfare effect of an increase in α is necessarily positive, given that the equilibrium innovation rate may be above the first-best level due to business stealing (Section 3.1). Note, however, that long-term contracts involve an inefficiency since the incumbent makes sales of an old technology to captive customers. Thus, even if an increase in the share of captive customers brings a socially excessive innovation rate closer to the first-best level, the waste effect dominates and aggregate welfare is reduced. (Indeed, observe that potential entrants, who directly suffer from the business-stealing effect, are necessarily better off when α increases.)

3.3. Predatory pricing

We next consider a model of predatory pricing, in which the entrant's probability of survival after its first production period is an increasing continuous function $\lambda(\pi_E)$ of its first-period profit. (This could be due to the entrant's financial constraints in an imperfect credit market, as in Bolton and Scharfstein [1990].) In this situation, the incumbent will be willing to price below c in the period following entry to increase the likelihood of forcing the entrant out of the market. To see this, consider first what the pricing equilibrium would be absent any antitrust constraint. In any such equilibrium, the entrant still wins, and consumers are indifferent between the two firms' products: the incumbent charges price p and the entrant charges price $p+\Delta$. This is an equilibrium if and only if p satisfies

$$p \leq c - [\lambda(p+\Delta-c) - \lambda(0)]\,(V_I - V_E) \leq p + \Delta.$$

The first inequality ensures that the incumbent prefers to lose at price p [rather than undercutting the price by ε, losing money on the sale, but increasing his chances of survival by $\lambda(p+\Delta-c) - \lambda(0)$]. The second inequality ensures that the entrant prefers to win at price $p+\Delta$. Assuming that $V_I - V_E > 0$, the middle expression is decreasing in p. Note also that the second inequality holds whenever $p \geq c - \Delta$ and the first inequality holds strictly at $p = c - \Delta$. Thus, at the highest equilibrium price p^* the first inequality binds, i.e.

$$p^* = c - [\lambda(p^*+\Delta-c) - \lambda(0)]\,(V_I - V_E)\,.$$

Note that $p^* \in (c - \Delta, c)$. We focus on the equilibrium in which the incumbent charges p^*, since this strategy for the incumbent weakly dominates charging any $p < p^*$.

Now consider an antitrust policy that imposes a price floor $\alpha < c$ on the incumbent. Suppose that the floor is binding, i.e., $p^* < \alpha$. In this case, $\pi_E(\alpha) = \alpha + \Delta - c$, $\pi_I(\alpha) = 0$, and $\pi_m(\alpha) = \Delta$: thus, a higher α raises $\pi_E(\alpha)$ upon entry, does not affect $\pi_I(\alpha)$ or $\pi_m(\alpha)$, and raises $\lambda(\alpha)$. If the policy only had an effect on π_E but not on λ, then, by (2.2), the policy would stimulate innovation. However, the policy also increases the entrant's probability of survival λ. Recall from the argument Subsection 2.4 that this effect also stimulates innovation. Thus, we conclude that a restriction on predatory pricing will stimulate innovation.[19]

As in the model of long-term contracts, an increase in α holding s fixed eliminates an inefficiency, here the inefficient loss of a new innovation. However, unlike the long-term contracting model, we cannot conclude that an increase in α necessarily raises aggregate welfare. To see one example in which welfare falls when α increases, suppose that the probability of survival $\lambda(\cdot)$ is constant at $\overline{\lambda}$ around $\alpha + \Delta - c$. Then a small increase in α will raise the price the entrant receives in his first period in the market (and lower consumers' payoffs in that period), have no effect on an entrant's survival probability, and will raise the level of R&D. Because the first effect is a pure transfer, the overall effect in welfare will be determined simply by whether we have too much or too little R&D given the survival rate $\overline{\lambda}$, which can go either way just as in Section 3.1.[20] By way of contrast, if we instead have a perfectly inelastic innovation supply, α affects aggregate welfare only through an increased probability of the entrant's survival, which unambiguously raises welfare whenever $\lambda(\cdot)$ is strictly increasing.

[19] In a more general model with differentiated products, predation would make both the entrant and incumbent lose money. Thus, increasing α would raise both firms' profits as well as the entrant's probability of survival, and so would again increase innovation.

[20] The reason we cannot use an argument like that leading to Proposition 3.2 is that the price increase has a direct negative effect on consumers plus the current incumbent; in contrast, in the long-term contracting model, all direct effects on the consumers plus the current incumbent were positive.

3.4. Voluntary deals

A simple implication of Proposition 2.1 is that deals between the entrant and incumbent that are advantageous to both firms are good for innovation. As an example, imagine that in our long-term contracting model the incumbent can license a new entrant's technology for serving his captive consumers. Specifically, assume that the incumbent is then able to make a take-it-or-leave-it offer to these captive buyers, offering to give them instead the entrant's better product for an additional payment of Δ. The incumbent and the entrant split the gain of $(1-\alpha)\Delta$ from the agreement. Under these assumptions, both π_I and π_E will increase, while π_m will be unaffected. As such, Proposition 2.1 tells us that the rate of innovation will increase if such deals are allowed. Moreover, using similar reasoning to that in Proposition 3.2, we can see that allowing such deals will also increase aggregate welfare.[21]

As another example, imagine that the incumbent and the entrant are allowed to collude in their pricing to free buyers in the period in which the entrant enters. Thus, the entrant now sells to these buyers at a price of 2Δ. We assume that the profit gain of $\alpha\Delta$ is split between the entrant and current incumbent through a side payment. In this case, π_E increases by $\alpha\Delta/2$, π_I increases from $(1-\alpha)(1-s)\Delta$ to $(1-\alpha)\Delta$ (free buyers no longer receive a surplus gain in the period of entry and therefore are willing to agree to a long-term contract at no discount), and π_E increases from $(1-\alpha)(1-s)\Delta+\alpha\Delta$ to Δ. Thus, again the rate of innovation increases. In this case, however, the welfare effects are not clear. We cannot use the same type of argument as in Proposition 3.2 to show that welfare increases because the direct effect of the change on the current incumbent plus consumers is negative. Indeed, observe that there is no direct efficiency effect arising from this collusive arrangement; it is merely a transfer from consumers to the firms. Thus, the sign of the effect on aggregate welfare is determined simply by whether we were initially

[21] As in the long-term contracting model, both the direct effect of the policy change and the indirect effect of the induced increase in innovation on the current incumbent plus consumers is positive. Since potential entrants must again be better off if the rate of innovation has increased, this implies that aggregate welfare has increased.

in a situation of over- or under-investment in R&D relative to the first-best.[22]

3.5. Uncertain innovation size and selection effects

Up to this point, we have considered models in which the nature of an innovation was fixed and antitrust policy could affect only the rate at which such innovations were discovered. In this section, we consider an extension of the long-term contracting model in which innovations are random and innovators must incur costs to bring them to market quickly. In such a setting, antitrust policy can affect not only the rate of discovery, but also the speeds with which different types of innovations make it to the market. Thus, antitrust policy also involves "selection effects." Intuitively, some innovations may bring only small benefits to their innovators, but may create large costs for the incumbents they replace. This may lead to circumstances in which more protective antitrust polices may retard innovation.

To explore this possibility, we consider an extension of the long-term contracting model in which a new innovator must pay $K > 0$ to enter the market immediately. If he does not incur this cost, he enters in the following period at no cost. We assume that the distribution of innovation sizes Δ is given by the cdf $F(\cdot)$ and for convenience we define $G(\Delta) \equiv 1 - F(\Delta)$.

To begin, observe that in this setting, if α is the share of free customers, a new innovator will enter immediately if and only if his innovation size Δ_E satisfies $\alpha\Delta_E \geq K$, or equivalently, $\Delta_E \geq \widehat{\Delta}(K, \alpha) \equiv \frac{K}{\alpha}$.

Consider now a consumer's decision of whether to accept a contract from an incumbent whose product's value is v_I and whose innovation size was Δ_I, when the innovation success rate is s and the cut-off type for immediate entry is $\widehat{\Delta}$. If the consumer accepts he gets $v_I - q_{t+1}$, while if he rejects he gets $(v_I - \Delta_I - c) + sG(\widehat{\Delta})$. Hence, the incumbent will charge $q_{t+1} = c + [1 - sG(\widehat{\Delta})]\Delta_I$.

[22] In a model with more general demand functions there would be an additional efficiency loss from the collusive deal because of increased pricing distortions.

This gives rise to the following profit functions:

$$
\begin{aligned}
\pi_m(\alpha, \Delta_I) &= [\alpha\Delta_I + (1-\alpha)(1-sG)\Delta_I] \\
&= [1-(1-\alpha)s(1-F)]\Delta_I \\
\pi_I(\alpha, \Delta_I) &= [\alpha F\Delta_I + (1-\alpha)(1-sG)\Delta_I] \\
&= [\alpha F + (1-\alpha)(1-sG)]\Delta_I \\
\pi_E(\alpha, \Delta_E) &= Max\{\alpha\Delta_E - K, 0\}
\end{aligned}
\tag{3.7}
$$

It is straightforward to see that condition (2.2) extends to the case of uncertain innovation, where now innovation increases (decreases) if

$$
\overline{\pi}'_E(\alpha) + \delta\left[\frac{(1-s)\overline{\pi}'_m(\alpha) + s\overline{\pi}'_I(\alpha)}{1-\delta(1-s)}\right] \geq (\leq) 0 \tag{3.8}
$$

at all feasible s, where the $\overline{\pi}$ functions are the expected profit functions (the expectation is taken with respect to the innovation size Δ). Since (3.7) gives us expected profit functions of

$$
\begin{aligned}
\overline{\pi}_m(\alpha) &= [1-(1-\alpha)s(1-F)]\overline{\Delta}_I \\
\overline{\pi}_I(\alpha) &= [\alpha F + (1-\alpha)(1-sG)]\overline{\Delta}_I \\
\overline{\pi}_E(\alpha) &= \int_{\frac{K}{\alpha}}^{\infty}(\alpha\Delta_E - K)f(\Delta_E)d\Delta_E,
\end{aligned}
$$

we have:

Proposition 3.3. *In the long-term (exclusives) model with random innovation size and costs of rapid implementation, restricting the use of long-term contracts increases (decreases) the rate of innovation s if*

$$\int_{\frac{K}{\alpha}}^{\infty} \Delta f(\Delta) d\Delta_E - \left(\frac{\delta}{1-\delta+\delta s}\right) s f(\frac{K}{\alpha}) \left(\frac{K}{\alpha^2}\right) \overline{\Delta}_I \geq (\leq) 0 \tag{3.9}$$

at all feasible s.

Substituting $\widehat{\Delta} = \frac{K}{\alpha}$ and rearranging, (3.9) can be written as

$$\left(\frac{\alpha}{f(\frac{K}{\alpha})\widehat{\Delta}}\right)\left(\frac{\int_{\widehat{\Delta}}^{\infty} \Delta f(\Delta) d\Delta_E}{\overline{\Delta}_I}\right) > \left(\frac{\delta s}{1-\delta+\delta s}\right), \tag{3.10}$$

This gives us the following result:

Corollary 3.4. *In the long-term (exclusives) model with random innovation size and costs of rapid implementation, restricting the use of long-term contracts increases the rate of innovation s if $f \approx 0$. It lowers the rate of innovation if $\alpha > 0$, the support of Δ is bounded with $f > \underline{f} > 0$ on this support, and $F(\widehat{\Delta}) \approx 1$.*

It is useful to decompose the effect of a change in α in the current model into two effects, the direct effect of the change in α holding the cut-off type $\widehat{\Delta}$ fixed, and the indirect effect of the change in $\widehat{\Delta}$.

Consider the first of these. Just as in the basic long-term (exclusive) contracting model of Section 3.2, the expected profit of a continuing incumbent, $s\overline{\pi}_I + (1-s)\overline{\pi}_m = [1 - s(1 - F(\widehat{\Delta})]\overline{\Delta}$, is unaffected by a change in α holding $\widehat{\Delta}$ fixed. On the other hand, the entrant's expected profit upon successful innovation, $\overline{\pi}_E$, continues to be increasing in α, just as in the basic model, although here as $F(\widehat{\Delta}) \to 1$, this effect also approaches zero.

Now consider the effect of a decrease in the cut-off type $\widehat{\Delta}$. By the envelope theorem, this has no effect on the expected profit of a successful innovator, $\overline{\pi}_E$, since the marginal type $\widehat{\Delta}$ who is entering is earning zero, but reduces the expected profit of a continuing incumbent, $[1 - s(1 - F(\widehat{\Delta})]\overline{\Delta}$.

To understand Corollary 3.4 observe that when $f(\widehat{\Delta}) \approx 0$, the indirect effect of a change in the cut-off type is of negligible importance since there is almost no change in the likelihood of a successful innovator entering immediately. In this case, the direct effect dominates and innovation increases with α, just as in the basic model. When $F(\widehat{\Delta})$ is close to 1, on the other hand, the direct effect on $\overline{\pi}_E$ approaches 0 and the indirect effect dominates, and so the innovation rate falls.

Of course, even when an increase in α causes the rate of innovation s to fall, successful innovations are more likely to come into the market quickly, since the cut-off type $\widehat{\Delta}$ decreases. Thus, the welfare effects of this change in innovation seem unclear. One can, however, establish the following result:

Proposition 3.5. *Aggregate welfare increases if s increases (which implies that $s[(1 - F) + \frac{\delta}{1-\delta}]$ increases). Aggregate welfare decreases if $s[(1 - F) + \frac{\delta}{1-\delta}]$ decreases (which implies that s decreases).*

Proof. See Appendix.

Observe that the discounted social value of a new innovation of size Δ is exactly $s[(1 - F) + \frac{\delta}{1-\delta}]\Delta$. Proposition 3.5 tells us that if s decreases in response to an increase in α sufficiently to make this value decline, we can be sure that aggregate welfare declines as well. To examine whether this is possible, we explore an example.

Example 3.6. *Let $c(\phi) = c\phi$ and $N = 1$ (so that $r = 1$). Then in any equilibrium (IS) takes the form*

$$W = c.$$

From (IB) we have

$$r\left(\frac{\theta + \delta c\phi}{1 - \delta + \delta(s + \phi r)}\right) = c,$$

where

$$\theta = \pi_E(1 - \delta + \delta s) + \delta\left[(1 - s)\overline{\pi}_m(\alpha) + s\overline{\pi}_I(\alpha)\right]. \tag{3.11}$$

So

$$\frac{r\theta}{1 - \delta + \delta s} = c.$$

Since $r = 1$ here, this tells us that s must adjust to keep $\frac{\theta}{1-\delta+\delta s}$ constant when α changes. Thus,

$$
\begin{aligned}
c &= Z(\alpha, s) \equiv \overline{\pi}_E(\alpha) + \left(\frac{\delta}{1-\delta+\delta s}\right)[(1-s)\overline{\pi}_m(\alpha) + s\overline{\pi}_I(\alpha)] \\
&= \int_{\frac{K}{\alpha}}^{\infty} (\alpha\Delta_E - K) f(\Delta) d\Delta_E + \left(\frac{\delta}{1-\delta+\delta s}\right)[1 - s(1 - F(\frac{K}{\alpha}))]\overline{\Delta} \\
&= \int_{\frac{K}{\alpha}}^{\infty} (\alpha\Delta_E - K) f(\Delta) d\Delta_E + \left\{\left(\frac{\delta}{1-\delta}\right)\overline{\Delta} - \left(\frac{\delta}{1-\delta+\delta s}\right)[(1 - F(\frac{K}{\alpha})) + \frac{\delta}{1-\delta}]s\overline{\Delta}\right\}.
\end{aligned}
$$

Observe that the expression in curly barckets makes sense: the continuation payoff starting in period $t+1$ of a successful innovatior in period t is the social PDV of his innovation less the social PDV of the first innovation to follow him. We now assume that $K \sim U[0, 1]$ and that $\alpha \in [K, 1]$. We also let $\delta = .9$ (a "period" is two years) and $K = 0.3$. Letting $s^*(\alpha)$ denote the equilibrium value of s given α, Figures 3.1-3.3 graph the values of $s^*(\alpha)$ and the per-period expected social value of innovation $s^*(\alpha)[(1-\delta)(1-\frac{K}{\alpha}) + \delta]$ for $c = 1$, $c = 1$, and $c = 3$. In each case, the solid line is $s^*(\alpha)$, while the dashed line is $s^*(\alpha)[(1-\delta)(1-\frac{K}{\alpha}) + \delta]$. In each case, welfare appears to have an interior local minimum, with the optimal policy either being a ban on long-term contracts ($\alpha = 1$), or unrestricted contracting ($\alpha = 0.3 = K$). The optimum is unrestricted contracting when $c = 0.1$, and is no long-term contracting at $c = 3$. It is less clear which is optimal when $c = 1$. It is also interesting to observe that if the share of captive customers is currently high, a move toward a less protective policy can be the best local change in policy, even when a full ban on long-term contracting is globally optimal.

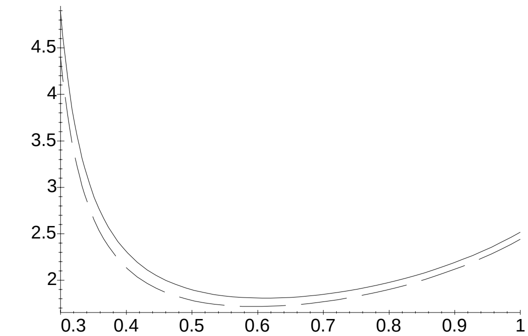

Figure 3.1: $c = 0.1$

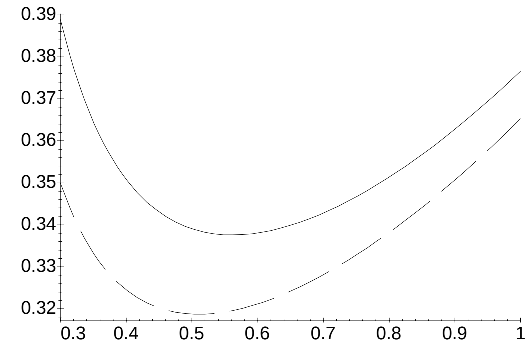

Figure 3.2: $c = 1$

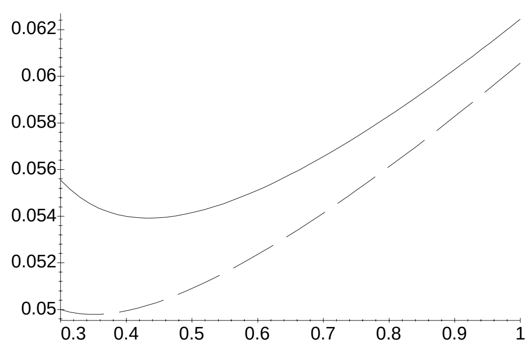

Figure 3.3: $c = 3$

The results for this model of random innovation size are related to those of O'Donohue et al.'s [1998] study of patent policy in a model of continuing innovation. In particular, O'Donohue et al. show that "leading patent breadth," the requirement that an innovation be at least a certain minimal amount better than the current leading technology to get a patent, can increase the rate of innovation and aggregate welfare. Here, an increased share of long-term contracts shifts upward the cut-off level of innovation that comes into the market rapidly. In this sense, changing the share of long-term contracts has effects like a change in the leading breadth of patent protection.[23]

This similarity between the effects of antitrust policy and patent policy raises the question of how optimal antitrust policy should be affected by the ability to also optimally set patent policy. While a full analysis is beyond our scope here, some insight can be gained by considering the introduction of a simple leading breadth policy into our model. Imagine, then, that we can also set directly a cut-off level Δ_C such that no innovation of size less than Δ_C can come into the market immediately. Suppose we start with an antitrust policy α which is less than 1 and an equilibrium cut-off level of $\widehat{\Delta}$. It is clear that nothing is changed if we set $\Delta_C = \widehat{\Delta}$. However, once we have done this, we change the effects of raising α. In particular, now an increase in α no longer has any effect on the set of innovations being immediately implemented; only the "direct effect" of an increase in α on profit levels remains, which we have seen causes innovation to increase. Moreover, this increase in innovation without any change in the set of innovations being immediately

[23] That said, the reason leading breadth has an effect in O'Donohue et al [1998] is quite different from here. Like our model, they posit a cost of implementing an innovation (in their case, at all rather than one period earlier as here), but in their model, the rate of innovation is exogeneous. Increasing leading patent breadth therefore necessarily reduces the number of innovations that can enter the market without infringing an incumbent's patent. They assume, however, that infringing innovations can be licensed to the current incumbent, who then implements them. Since increased breadth increases the length of time until the incumbent is displaced by a noninfringing innovation, the incumbent is more willing to license infringing innovations the larger is leading breadth. In the limit, as leading breadth grows infinity large, the incumbent "owns the entire quality ladder" and implements exactly the first-best set of innovations.

In contrast, in our model the rate of innovation is endogeneous. Leading breadth can have a positive welfare effect in a model such as ours for a different reason than that in O'Donohue et al.: because innovations of small size generate little profit for an entrant but can destroy large profit levels for an incumbent, R&D effort can be stimulated by excluding some small innovations from the market.

implemented necessarily raises welfare. Hence, the optimal antitrust policy when patent policy is available sets $\alpha = 1$. Intuitively, while antitrust policy can be used to prevent small innovations from coming to market, it does this only at the cost of introducing inefficiency when larger innovations come to market. Patent policy can achieve the same objective without this inefficiency, at least if innovation sizes are verifiable so that a leading patent breadth policy can be implemented.[24]

4. Incumbent Investment [incomplete]

The analysis above imposed the strong restriction that only potential entrants engaged in R&D. Although useful for gaining insight, this assumption is clearly not respresentative of most settings of interest. In this section, we explore how our conclusions are affected when incumbent firms may also engage in R&D.

Allowing incumbent firms to engage in R&D has the potential to considerably complicate the analysis. In particular, once we allow for incumbent investment, we need in general to introduce a state space to keep track of the incumbent's current lead over the potential entrants. In general, the rates of R&D investment by the incumbent and its challengers may be state dependent (see, for example, Aghion et. al. [2001]).

To date we have focused on two special cases in which R&D strategies are nonetheless stationary. Although clearly restrictive, these two models do have the virtue of capturing two distinct motives for incumbent R&D: (i) preventing displacement by an entrant, and (ii) increasing the flow of profits until displacement by increasing the lead over the previous incumbent.

[24] A leading breadth policy could in principle also be implemented indirectly, by requiring an innovator to pay a fee to gain access to the market, as in Llobet et al. [2000].

4.1. R&D to prevent displacement

As earlier, we focus on the case of a fixed number of firms N. The only change from the model of Section 2 is that the incumbent may now do R&D. We denote the levels of R&D for the incumbent and a potential entrant by ϕ_I and ϕ_E respectively, and the respective cost functions by $c_I(\phi_I)$ and $c_E(\phi_E)$ (we allow for the fact that the cost of achieving a discovery may differ between the incumbent and the potential entrants). In this first model, we assume that if the leading quality level in period t is j_t, then quality level $j_t - 1$ is freely available to all potential producers. That is, it enters the public domain. Thus, the incumbent never has a lead greater than one step on the ladder. Thus, the only reason for an incumbent to do R&D is to try to get the patent on the next innovation in cases where at least one potential entrant has made a discovery – that is, to prevent its displacement. To capture this in the simplest possible way, we assume that the incumbent gets the patent whenever it makes the discovery; that is, that the incumbent wins all "ties".[25] With these assumptions, we need not keep track of any states, and there is a stationary equilibrium.

Denoting by V_I and V_E the values of the incumbent and a potential entrant, we now have:

$$V_I = \pi_m(\alpha) + \delta V_I + s(\phi_E, N)(1-\phi_I)\{\pi_I(\alpha) - \pi_m(\alpha) + \delta(V_I - V_E)\} - c_I(\phi_I) \tag{4.1}$$

$$V_E = \delta V_E + \phi_E r(\phi_E, N)(1-\phi_I)\{\pi_E(\alpha) + \delta(V_I - V_E)\} - c_E(\phi_E). \tag{4.2}$$

[25] In the usual sort of (Poisson) continuous-time model considered in the R&D literature (see, e.g., Lee and Wilde [1980], Reinganum [1989], and Grossman and Helpman [1991]), the probability of ties is zero, and so one might worry that our formulation here is dependent on a merely technical feature of the discrete-time set-up. Indeed, in such a model, the incumbent would do no R&D here. However, the usual continuous-time model relies on the implicit assumption that following an innovation, all firms reorient their R&D activity immediately to the next technology level. If we were to instead use a continuous-time model in which there is a fixed time period after a rival's success before which R&D for the next technology level cannot be successful, then we would get effects that parallel those in our discrete-time model (where the discount factor δ reflects how quickly R&D activity can be reoriented to the next technology level.) Thus, our discrete-time formulation captures an arguably realistic feature of the economics of R&D.

Letting

$$W_I \equiv s(\phi_E, N)\left[\pi_m(\alpha) - \pi_I(\alpha) + \delta(V_I - V_E)\right] \tag{4.3}$$

and

$$W_E \equiv r(\phi_E, N)(1-\phi_I)\{\pi_E(\alpha) + \delta(V_I - V_E)\}, \tag{4.4}$$

we have $\phi_i \in \arg\max_{\psi_i \in [0,1]} \psi_i W_i - c_i(\psi_i)$ for $i = I, E$.. Solving (4.1) and (4.2) for $(V_I - V_E)$ and substituting we get (suppressing arguments of functions)

$$\begin{aligned} W_I &= \left(\frac{s}{D}\right)\{\pi_m - (1-\delta)\pi_I + \delta(1-\phi_I)r\phi_E(\pi_m - \pi_I - \pi_E) + \delta(c_E - c_I)\} \\ W_E &= \left(\frac{r(1-\phi_I)}{D}\right)\{\delta\pi_m + (1-\delta)\pi_E - \delta(1-\phi_I)s(\pi_m - \pi_I - \pi_E) + \delta(c_E - c_I)\}, \end{aligned}$$

where $D \equiv 1 - \delta + \delta[1 - \phi_I(s + r\phi_E)]$.

In this setting where both the incumbent and potential entrants can do R&D we can distinguish between the direct effects of a change in the policy α and the indirect effects. For the incumbent, the former captures the change in its R&D incentives holding fixed the R&D of potential entrants ϕ_E, and has the same sign as the change in W_I caused by the change in α holding (ϕ_I, ϕ_E) fixed. Similarly, the direct effect for the potential entrants has the same sign as the change in W_E caused by the change in α holding (ϕ_I, ϕ_E) fixed.

Proposition 4.1. *In the model of incumbent R&D to prevent displacement, the direct effect of a more protective antitrust policy (an increase in α) on incumbent R&D is positive if and only if*

$$-\frac{\pi_I'(\alpha)}{\pi_E'(\alpha)} \geq \frac{\delta(1-\phi_I)r\phi_E}{(1-\delta) + \delta(1-\phi_I)r\phi_E} - \left(\frac{\pi_m'(\alpha)}{\pi_E'(\alpha)}\right)\left(\frac{1 + \delta(1-\phi_I)r\phi_E}{(1-\delta) + \delta(1-\phi_I)r\phi_E}\right) \tag{4.5}$$

and is positive on potential entrant R&D if and only if

$$\left[1+\frac{1-\delta}{\delta s(1-\phi_I)}\right]+\left(\frac{1-s(1-\phi_I)}{s(1-\phi_I)}\right)\left(\frac{\pi'_m(\alpha)}{\pi'_E(\alpha)}\right)\geq-\frac{\pi'_I(\alpha)}{\pi'_E(\alpha)}. \tag{4.6}$$

A few observations can be made about Proposition 4.1. First, note that the direct effects of a more protective antitrust policy on incumbent and potential entrant innovation can never both be negative provided that $\pi'_m(\alpha) \geq 0$, since in that case the righthand side of (4.5) is less than 1, while the lefthand side of (4.6) is greater than 1. More strikingly, when $\pi'_m(\alpha) \geq 0$ and $-\frac{\pi^{\theta}_I(\alpha)}{\pi^{\theta}_E(\alpha)} \approx 1$ both direct effects are positive. Intuitively, when $\pi'_I(\alpha) \leq 0$, a more protective antitrust policy can encourage incumbent innovation when innovation is done to avoid displacement because it reduces the incumbent's profits when entry occurs, thus making avoiding that outcome all the more desirable for the incumbent [the other effect, which leads to the ambiguity in (4.5) in general, is that it also reduces the value of $(V_I - V_E)$]. Similarly, when $\pi'_m(\alpha) \geq 0$ and $\pi'_I(\alpha) \leq 0$, if $\delta(1-\phi_I)\phi_E \approx 0$ (e.g., if the rate of either incumbent innovation or time discount is very high or the rate of entrant innovation is very low) then the incumbent's direct effect is necessarily positive.

The direct effects are not determinative, however, of the overall change in equilibrium innovation rates, because there are interactions between the R&D levels of the incumbent and potential entrants since the level of ϕ_i in general affects the value W_j $(i \neq j;\, j = I, E)$. It can be seen from (4.3) and (4.4) that when $(\pi_m - \pi_I - \pi_E) \approx 0$, the level of incumbent innovation increases in ϕ_E, and the level of potential entrant innovation decreases in ϕ_I. When this is so, and the direct effects are both positive, we know that the incumbent's innovation rate ϕ_I must increase with an increase in α.[26]

[26] More generally, when $(\pi_m - \pi_I - \pi_E) \geq 0$ the incumbent's R&D level is increasing in ϕ_I, although the direction of the indirect effect on potential entrants' R&D is in this case ambiguous.

4.2. R&D to increase profit flows

We next consider a model in which rivals do not get access to the second best technology when the incumbent innovates. Thus, the incumbent can increase its flow of profits by innovating, until the time when it is displaced. Specifically, let k denote the number of steps that the incumbent is ahead of its nearest rival (this is our state variable). The variable k affects the incumbent's profit flow when entry does not occur, which we now denote by $\pi_m(k,\alpha)$ (it does not affect either π_I or π_E). We now make two assumptions that will imply that there is an equilibrium in which the R&D levels of the incumbent and potential entrants do not depend upon k. Specifically, we assume that $\pi_m(k,\alpha) = k\pi_m(k)$ and that an entrant gets the patent whenever at least one entrant has made a discovery.

It is clear that there is a solution in which potential entrant R&D ϕ_E and value V_E are stationary. To begin, we allow that the incumbent's R&D and value functions may depend on k: ϕ_I^k and V_I^k. In this case, we can write the value equations as

$$\begin{aligned} V_I^k &= k\pi_m + \delta V_I^k + s(\phi_E, N)\{(\pi_I - k\pi_m) + \delta[V_E - V_I^k]\} \\ &\quad + \phi_I^k[1 - s(\phi_E, N)]\{[(k+1)\pi_m - k\pi_m) + \delta[V_I^{k+1} - V_I^k]\} - c_I(\phi_I^k), \end{aligned} \tag{4.7}$$

for $k \geq 1$, and

$$V_E = \delta V_E + \phi_E r(\phi_E, N)\left[\pi_E + \delta\left(V_I(1) - V_E\right)\right] - c_E\left(\phi_E\right), \tag{4.8}$$

while the equilibrium innovation rates satisfy

$$\phi_I^k \in \arg\max_{\psi_I^k \in [0,1]} \psi_I^k[1 - s(\phi_E, N)]\{[(k+1)\pi_m - k\pi_m) + \delta[V_I(k+1) - V_I(k)]\} - c_I(\psi_I^k), \tag{4.9}$$

$$\phi_E \in \arg\max_{\psi_E \in [0,1]} \psi_E r(\phi_E, N)\left[\pi_E + \delta\left(V_I(1) - V_E\right)\right] - c_E\left(\psi_E\right). \tag{4.10}$$

Now observe from (4.9) that ϕ_I^k will be independent of k if the difference $V_I(k+1) - V_I(k)$

is. Using (4.7) for k and $k+1$ we see that

$$\begin{aligned}(1-\delta)\left[V_I^{k+1}-V_I^k\right] &= \pi_m - s(\phi_E,N)\{\pi_m+\delta[V_I^{k+1}-V_I^k]\} \\ &+[1-s(\phi_E,N)]\left[\phi_I^{k+1}\{\pi_m+\delta[V_I^{k+2}-V_I^{k+1}]\}-\phi_I^k\{\pi_m+\delta[V_I^{k+1}-V_I^k]\}\right] \\ &-c_I(\phi_I^{k+1})+c_I(\phi_I^k).\end{aligned}$$

Hence, if ϕ_I^k is independent of k, we have

$$\left[V_I^{k+1}-V_I^k\right]=\frac{\pi_m(1-s)}{1-\delta+\delta s},$$

which is indepent of k. Hence, there exists an equilibrium in which the incumbent's innovation rate is independent of k, $\phi_I \equiv \phi_I^k$, and [specializing (4.9) satisfies

$$\phi_I \in \arg\max_{\psi_I\in[0,1]} \psi_I(1-s)\left[\pi_m+\delta\left(\frac{\pi_m(1-s)}{1-\delta+\delta s}\right)\right]-c_I(\psi_I)=\arg\max_{\psi_I\in[0,1]} \psi_I\pi_m\left[\frac{1-s}{1-\delta+\delta s}\right]-c_I(\psi_I). \tag{4.11}$$

Thus,

$$W_I=\pi_m\left[\frac{1-s}{1-\delta+\delta s}\right]. \tag{4.12}$$

We next solve for W_E. Subtracting the expression for V_E from that for V_I^1 we have (omitting arguments of functions for notational simplicity)

$$[V_I^1-V_E][1-\delta+\delta(s+\phi_E r)]=\pi_m+s(\pi_I-\pi_m)+\phi_I\pi_m\left[\frac{1-s}{1-\delta+\delta s}\right]-\phi_E r\pi_E-(c_I-c_E).$$

Thus,

$$W_E=\left[\frac{r}{1-\delta+\delta(s+\phi_E r)}\right]\left\{\pi_E[1-\delta+\delta s]+\delta s\pi_I+\delta(1-s)\left[\frac{2-\delta(1-s)}{1-\delta(1-s)}\right]\pi_m-\delta(c_I-c_E)\right\}. \tag{4.13}$$

Examing (4.12) and (4.13) we have

Proposition 4.2. *In the model of incumbent R&D to increase profit flows, the direct effect on incumbent R&D of a more protective antitrust policy (an increase in α) has the same sign as $\pi_m'(\alpha)$, while the direct effect on potential entrant R&D is positive if and only if*

$$\left[1+\frac{1-\delta}{\delta s}\right]+\left(\frac{1-s}{s}\right)\left(\frac{\pi_m'(\alpha)}{\pi_E'(\alpha)}\right)\geq -\frac{\pi_I'(\alpha)}{\pi_E'(\alpha)}. \tag{4.14}$$

Once again we can get both direct effects to be positive; indeed, this is certain to be the case if the increase in α raises bot hthe monopoly profit π_m and the joint profit upon entry $\pi_I+\pi_E$. Considering now the indirect effects, we see from (4.13) that the level of incumbent R&D has no indirect effect on ϕ_E, while from (4.12) increases in the level of potential entrant R&D (and, hence, s) reduce the level of ϕ_I.

5. Conclusion

To be added.

6. Appendix

Proof of Proposition 3.5: As before, a potential entrant's value increases if and only if s increases. We now consider the payoffs of the current incumbent, consumers, and the sum of the current incumbent and consumers.

Current Incumbent: We look at the expected continuation payoff for an incumbent of type Δ_I starting in period $\tau+1$. As before this takes the form $\delta[s_0V_E+(1-s_0)V_I]$ where now

$$\begin{aligned}(1-\delta+\delta s)V_I &= [(1-s)\pi_m+s\pi_I]+\delta s V_E\\ &= [1-s(1-F)]\Delta_I+\delta s V_E.\end{aligned}$$

Consumers: Consider the consumers' expected continuation surplus starting in period $\tau+1$. Note that the first innovation (beginning in period τ) gives a consumer gain of Δ_I, after that innovation give an expected gain of $\overline{\Delta}$. This can be written as

$$
\begin{aligned}
& \delta(1-s_0)\left[\begin{array}{c}
\left\{(v_\tau - c - \Delta_I) + s\Delta_I[(1-F)+\frac{\delta}{1-\delta}]\right\}+ \\
\delta\left\{\begin{array}{c}
(v_\tau - c - \Delta_I) + (1-s)s\Delta_I[(1-F)+\frac{\delta}{1-\delta}]+ \\
[1-(1-s)]s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]
\end{array}\right\}+ \\
\delta^2\left\{\right\}...
\end{array}\right] \\
& +\delta s_0\left[\begin{array}{c}
\left\{(v_\tau - c) + s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]\right\}+ \\
\delta\left\{(v_\tau - c) + s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]\right\}+ \\
\delta^2\left\{\right\}...
\end{array}\right] \\
= \; & \delta\left[\frac{(v_\tau - c - (1-s_0)\Delta_I)}{1-\delta}\right]+ \\
& \delta(1-s_0)\left[\begin{array}{c}
s\Delta_I(1-F)+ \\
\delta(1-s)s\Delta_I(1-F)+ \\
\delta^2(1-s)^2 s\Delta_I(1-F)+...
\end{array}\right]+ \\
& \delta(1-s_0)\left[\begin{array}{c}
s\Delta_I\frac{\delta}{1-\delta}+ \\
\delta(1-s)s\Delta_I\frac{\delta}{1-\delta}+ \\
\delta^2(1-s)^2 s\Delta_I\frac{\delta}{1-\delta}+...
\end{array}\right]+ \\
& \delta(1-s_0)\left[\begin{array}{c}
\delta[1-(1-s)]s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]+ \\
\delta^2[1-(1-s)^2]s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]+ \\
\delta^3[1-(1-s)^3]s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]+...
\end{array}\right]+
\end{aligned}
$$

$$
\begin{aligned}
& \delta s_0 \begin{bmatrix} s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]+ \\ \delta s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]+ \\ \delta^2 s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]+... \end{bmatrix} \\
= \; & \delta \left[\frac{(v_0-c-(1-s_0)\Delta_I)}{1-\delta}\right]+ \\
& \delta(1-s_0)s\Delta_I(1-F)\left(\frac{1}{1-\delta(1-s)}\right)+ \\
& \delta(1-s_0)s\Delta_I\frac{\delta}{1-\delta}\left(\frac{1}{1-\delta(1-s)}\right)+ \\
& \delta(1-s_0)s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]\delta\left(\frac{1}{1-\delta}-\frac{1-s}{1-\delta(1-s)}\right)+ \\
& \delta s_0 s\overline{\Delta}[(1-F)+\frac{\delta}{1-\delta}]\frac{1}{1-\delta} \\
= \; & \delta \left[\frac{(v_0-c-(1-s_0)\Delta_I)}{1-\delta}\right]+ \\
& \delta(1-s_0)\left(\frac{s(1-F)}{1-\delta(1-s)}\right)\Delta_I+ \\
& (1-s_0)\Delta_I\frac{\delta}{1-\delta}\left(\frac{\delta s}{1-\delta+\delta s}\right)+ \\
& \delta(1-s_0)\left(\frac{\delta}{1-\delta}\right)\left(\frac{\delta s}{1-\delta+\delta s}\right)s[(1-F)+\frac{\delta}{1-\delta}]\overline{\Delta}+ \\
& \delta s_0\left(\frac{1}{1-\delta}\right)s[(1-F)+\frac{\delta}{1-\delta}]\overline{\Delta} \\
= \; & \delta \left[\frac{(v_0-c-(1-s_0)\Delta_I)}{1-\delta}\right]+ \\
& \delta(1-s_0)\left(\frac{s(1-F)}{1-\delta(1-s)}\right)\Delta_I+ \\
& (1-s_0)\Delta_I\frac{\delta}{1-\delta}\left(\frac{\delta s}{1-\delta+\delta s}\right)+ \\
& \delta\left\{(1-s_0)\left(\frac{\delta}{1-\delta}\right)\left(\frac{\delta s}{1-\delta+\delta s}\right)+s_0\left(\frac{1}{1-\delta}\right)\right\}s[(1-F)+\frac{\delta}{1-\delta}]\overline{\Delta}.
\end{aligned}
$$

Sum of Incumbent and Consumers: Adding these two expressions together we get

$$
\begin{aligned}
= \; & \delta\left[\frac{(v_\tau - c - (1-s_0)\Delta_I)}{1-\delta}\right] + \\
& \delta(1-s_0)\left(\frac{1}{1-\delta+\delta s}\right)\Delta_I + \\
& (1-s_0)\Delta_I \frac{\delta}{1-\delta}\left(\frac{\delta s}{1-\delta+\delta s}\right) + \\
& \delta\left\{(1-s_0)\left(\frac{\delta}{1-\delta}\right)\left(\frac{\delta s}{1-\delta+\delta s}\right) + s_0\left(\frac{1}{1-\delta}\right)\right\} s[(1-F) + \frac{\delta}{1-\delta}]\overline{\Delta} \\
= \; & \delta\left[\frac{(v_\tau - c - (1-s_0)\Delta_I)}{1-\delta}\right] + \\
& (1-s_0)\frac{\delta}{1-\delta+\delta s}\left(1 + \frac{\delta s}{1-\delta}\right)\Delta_I + \\
& \delta\left\{(1-s_0)\left(\frac{\delta}{1-\delta}\right)\left(\frac{\delta s}{1-\delta+\delta s}\right) + s_0\left(\frac{1}{1-\delta}\right)\right\} s[(1-F) + \frac{\delta}{1-\delta}]\overline{\Delta} \\
= \; & \delta\left[\frac{(v_\tau - c - (1-s_0)\Delta_I)}{1-\delta}\right] + \\
& (1-s_0)\left(\frac{\delta}{1-\delta}\right)\Delta_I + \\
& \delta\left\{(1-s_0)\left(\frac{1}{1-\delta}\right)\left(\frac{\delta s}{1-\delta+\delta s}\right) + s_0\left(\frac{1}{1-\delta}\right)\right\} s[(1-F) + \frac{\delta}{1-\delta}]\overline{\Delta} \\
= \; & (v_\tau - c)\left(\frac{\delta}{1-\delta}\right) + \\
& \delta\left\{(1-s_0)\left(\frac{\delta}{1-\delta}\right)\left(\frac{\delta s}{1-\delta+\delta s}\right) + s_0\left(\frac{1}{1-\delta}\right)\right\} s[(1-F) + \frac{\delta}{1-\delta}]\overline{\Delta},
\end{aligned}
$$

from which follows the result. ■

Remark 1. *The way to think about the expression above is the following: for sure, the initial incumbent and the consumers together get* $(v_0 - c)$ *in every period. The first subsequent innovation and event merely is a transfer from the inital incumbent to the consumers. Starting with the second innovation consumers get a further gain (and the inital incumbent is unaffected). If an innovation happens in period* τ *which is true with probability* s_0*, the expected PDV of this is* $\left(\frac{1}{1-\delta}\right) s[(1-F) + \frac{\delta}{1-\delta}]\overline{\Delta}$ *since every period* *from $\tau+1$ on we have a probability s of adding $[(1-F)+\frac{\delta}{1-\delta}]\overline{\Delta}$. If, instead, there is no innovation in period τ which is true with probability $(1-s_0)$, then we must wait until the 2nd innovation happens to start recording these gains. The expected PDV of this starting in period $\tau+1$ is then*

$$\begin{aligned}
&= 0+ \\
&\quad \delta[s\cdot s(1-F+\frac{\delta}{1-\delta})\overline{\Delta}]+ \\
&\quad \delta^2[(1-(1-s)^2)\cdot s(1-F+\frac{\delta}{1-\delta})\overline{\Delta}]+ \\
&\quad \ldots \\
&= s(1-F+\frac{\delta}{1-\delta})\overline{\Delta}\left\{\frac{\delta}{1-\delta}-\frac{\delta(1-s)}{1-\delta(1-s)}\right\}.
\end{aligned}$$

The sum of these two expressions is exactly the last line above. The inital incumbent plus consumers' joint value is increasing in $s(1-F+\frac{\delta}{1-\delta})\overline{\Delta}]$ because this is the expected value in each period to the consumer of innovation once one subsequent innovation has occurred; it is increasing in s because this makes the first subsequent innovation arrive sooner in a probabilistic sense.

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